16 TUBULAR JOINTPLASTICITY FORMULATION

This Section presents the theoretical basis of a plasticity model derived for joint modelling. Results from preliminary studies with a similar (simpler) formulation are shown in Section 2.4. Although the mathematics may seem quite involved, the main message is that it has been possible to derive a theoretically consistent plasticity formulation for the problem at hand, and that the formulation seems to work. It is also worth pointing out that a similar mathematical complexity is required for modelling material yielding/hardening behaviour in nearly all general purpose FE programs.

Details can be then left to the mathematically inclined...

16.1 General

Material nonlinearities are modelled by yield hinges introduced in the joint elements. The behaviour of the hinges is governed by plastic flow theory, according to an isotropic or a kinematic hardening model. Associated flow is assumed, with plastic potentials defined by interaction formulas for the element cross-section. The model is formulated in force-space, i.e. it relates plastic displacements and rotations to section forces and moments.

The novel aspects in the proposed approach is that:

i. the hardening behaviour for each force component is directly determined by an input P-* curve.

ii. each force component follows an independent hardening rule (given by the independent P, M_{ipb} and M_{opb} curves), resulting in a countinously changing shape of the yield surface.

16.2 Plastic interaction function

The plastic interaction function may be given by (3.1). Here, the capacity equations of the API and HSE codes are given together with a general, user-defined plastic potential defined in USFOS. For the user-defined capacity formulation, the shape of the plastic potential is given by the "()-parameters.

$$
' \; ' \begin{cases} \cos\left(\frac{B}{2}\frac{N}{N_U}\right) \,\&\, \sqrt{\left(\frac{M_{ipb}}{M_{ipb,U}}\right)^2 \,\%\, \left(\frac{M_{opb}}{M_{opb,U}}\right)^2} & \text{, API} \\ \left|\frac{N}{N_U}\right| \,\%\, \left(\frac{M_{ipb}}{M_{ipb,U}}\right)^2 \,\%\, \left|\frac{M_{opb}}{M_{opb,U}}\right| \,\&\, 1 & \text{, HSE} \\ \left(\frac{N}{N_U}\right)^{"1} \,\%\, \left(\left(\frac{M_x}{M_{x,U}}\right)^{"2} \,\%\, \left(\frac{M_{ipb}}{M_{ipb,U}}\right)^{"3} \,\%\, \left(\frac{M_{opb}}{M_{opb,U}}\right)^{"4} \right)^{"5} \,\&\, 1 & \text{, User defined} \end{cases} \tag{16.1}
$$

16.3 Elastic-Perfectly Plastic Model

The yield condition is represented by an interaction function between axial force, in-plane bending and out-of-plane bending.

$$\Gamma = f\left(\frac{N}{N_U}, \frac{M_{ipb}}{M_{ipb,U}}, \frac{M_{opb}}{M_{opb,U}}\right) - 1 = 0 \tag{16.2}$$

N, M_{ipb}, M_{opb} etc. are the joint forces and N_U, $M_{ipb,U}$, $M_{opb,U}$ are the joint capacities for each force component. $\Gamma = 0$ represents full plastification of the cross section. $\Gamma = -1$ is the initial value of a stress-free cross section. In principle, a state of forces characterized by $\Gamma > 0$ is illegal.

The flow rule for associated flow is given by

$$\Delta \boldsymbol{v}^P = \begin{bmatrix} \boldsymbol{g}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{g}_2 \end{bmatrix} \begin{bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \end{bmatrix} = \boldsymbol{G}\,\Delta\boldsymbol{\lambda} \tag{16.3}$$

stating that plastic displacement increments are normal to the cross sectional yield surface, Γ, multiplied by a scalar, $\Delta\lambda$. The surface normal is given by

$$\boldsymbol{g}_i^T = \frac{\partial \Gamma}{\partial \boldsymbol{S}_i} = \left[\frac{\partial \Gamma}{\partial N}, \frac{\partial \Gamma}{\partial M_{ipb}}, \frac{\partial \Gamma}{\partial M_{opb}}\right]_i \tag{16.4}$$

and index i refers to beam end 1 and beam end 2.

The consistency rule is defined such that the state of forces move from one legal plastic state to another plastic state, following the yield surface so that $\Gamma = 0$. For an elastic - perfectly plastic material model, this can be expressed as

$$\begin{aligned} \Delta\Gamma &= \frac{\partial \Gamma}{\partial N}\Delta N + \frac{\partial \Gamma}{\partial M_{ipb}}\Delta M_y + \frac{\partial \Gamma}{\partial M_{opb}}\Delta M_z \\ &= \boldsymbol{g}^T \Delta \boldsymbol{S}_i \\ &= 0 \end{aligned} \tag{16.5}$$

When both nodes are considered, (3.5) takes the form $\boldsymbol{G}^T \Delta\boldsymbol{S} = \boldsymbol{0}$

Elasto-plastic Stiffness Matrix

The elastic stiffness expression for the joint element is expressed as

$$\Delta\boldsymbol{S} = \boldsymbol{K}_E\,\Delta\boldsymbol{v}^E \tag{16.6}$$

The total displacement increment is separated into elastic and plastic components

$$\Delta\boldsymbol{v} = \Delta\boldsymbol{v}^E + \Delta\boldsymbol{v}^P \tag{16.7}$$

and the stiffness equation is expressed as

$$\begin{aligned} \Delta\boldsymbol{S} &= \boldsymbol{K}_E(\Delta\boldsymbol{v} - \Delta\boldsymbol{v}^P) \\ &= \boldsymbol{K}_E\,\Delta\boldsymbol{v} - \boldsymbol{K}_E\,\boldsymbol{G}\,\Delta\boldsymbol{\lambda} \end{aligned} \tag{16.8}$$

when the flow rule (3.3) is introduced.

Pre-multiplying with $\boldsymbol{G}^T$, the right-hand side takes the form of the consistency rule (3.5)

$$
\begin{aligned}
\boldsymbol{G}^T \Delta \boldsymbol{S} &= \boldsymbol{G}^T \boldsymbol{K}_E \Delta \boldsymbol{v} - \boldsymbol{G}^T \boldsymbol{K}_E \boldsymbol{G} \Delta \lambda \\
&= \boldsymbol{0}
\end{aligned}
\tag{16.9}
$$

and the plastic increment can be solved

$$
\Delta \lambda = \left(\boldsymbol{G}^T \boldsymbol{K}_E \boldsymbol{G} \right)^{-1} \left(\boldsymbol{G}^T \boldsymbol{K}_E \Delta \boldsymbol{v} \right) \tag{16.10}
$$

Substituting $\Delta \lambda$ back into (3.8), the elasto-plastic stiffness of the element becomes

$$
\begin{aligned}
\Delta \boldsymbol{S} &= \boldsymbol{K}_E \Delta \boldsymbol{v} - \boldsymbol{K}_E \boldsymbol{G} \Delta \lambda \\
&= \boldsymbol{K}_E \Delta \boldsymbol{v} - \boldsymbol{K}_E \boldsymbol{G} \left(\boldsymbol{G}^T \boldsymbol{K}_T \boldsymbol{G} \right)^{-1} \boldsymbol{G}^T \boldsymbol{K}_E \Delta \boldsymbol{v} \\
&= \left(\boldsymbol{K}_E - \boldsymbol{K}_E \boldsymbol{G} \left(\boldsymbol{G}^T \boldsymbol{K}_E \boldsymbol{G} \right)^{-1} \boldsymbol{G}^T \boldsymbol{K}_E \right) \Delta \boldsymbol{v} \\
&= \boldsymbol{K}_{EP} \Delta \boldsymbol{v}
\end{aligned}
\tag{16.11}
$$

16.4 Strain Hardening Model

At each state of plastic deformation at the hinge, there exists a unique capacity surface in force space given by

$$
= f\left(\frac{N}{N_0 R_N}, \frac{M_{ipb}}{M_{ipb,0} R_{ipb}}, \frac{M_{opb}}{M_{opb,0} R_{opb}} \right) - 1 = 0 \tag{16.12}
$$

where N_0, $M_{ipb,0}$, $M_{opb,0}$ are "elastic" joint capacities and R_k are hardening functions, expressed as a function of the plastic deformations for each force degree of freedom.

$$
\Delta \boldsymbol{R} = \boldsymbol{k}_h \Delta \boldsymbol{v}^P = \boldsymbol{k}_h \boldsymbol{g}_S \Delta \lambda \tag{16.13}
$$

In the context of joint modelling, the hardening function $\boldsymbol{R}(\boldsymbol{v}^P)$ can be directly derived from the nonlinear P-δ and M-θ curves. The "linear" part of the curves is extracted as $\boldsymbol{v}^E$; the remaining part of the curve is included as "hardening", where the degree of hardening is directly given by the plastic deformation $\boldsymbol{v}^P$ associated with each degree of freedom.

The consistency rule now takes the form

$$
\begin{aligned}
\Delta\Phi &= \frac{\partial\Phi}{\partial \boldsymbol{S}}\Delta S + \frac{\partial\Phi}{\partial R}\Delta R \\
&= \frac{\partial\Phi}{\partial N}\Delta N + \frac{\partial\Phi}{\partial M_{ipb}}\Delta M_{ipb} + \frac{\partial\Phi}{\partial M_{opb}}\Delta M_{opb} \\
&+ \frac{\partial\Phi}{\partial R_N}\Delta R_N + \frac{\partial\Phi}{\partial R_{ipb}}\Delta R_{ipb} + \frac{\partial\Phi}{\partial R_{opb}}\Delta R_{opb} \\
&= \boldsymbol{g}_S^T \Delta \boldsymbol{S}_i + \boldsymbol{g}_R^T \boldsymbol{k}^h \boldsymbol{g}_S \Delta\lambda \\
&= 0
\end{aligned}
\tag{16.14}
$$

When both nodes in the element are considered, (3.14) takes the form $\boldsymbol{G}_S^T \Delta \boldsymbol{S} + \boldsymbol{G}_R^T \Delta \boldsymbol{K}_H \boldsymbol{G}_S \Delta\lambda = \boldsymbol{0}$
The stiffness equation is again expressed by (3.8):

$$
\begin{aligned}
\Delta \boldsymbol{S} &= \boldsymbol{K}_E\,(\Delta \boldsymbol{v} - \Delta \boldsymbol{v}^P) \\
&= \boldsymbol{K}_E \Delta \boldsymbol{v} - \boldsymbol{K}_E \boldsymbol{G}_S \Delta\lambda
\end{aligned}
\tag{16.15}
$$

Pre-multiplying with $\boldsymbol{G}^T$ and combining with (3.14), the right-hand side now takes the form

$$
\begin{aligned}
\boldsymbol{G}_S^T \Delta \boldsymbol{S} &= \boldsymbol{G}_S^T \boldsymbol{K}_E \Delta \boldsymbol{v} - \boldsymbol{G}_S^T \boldsymbol{K}_E \boldsymbol{G}_S \Delta\lambda \\
&= -\boldsymbol{G}_R^T \boldsymbol{K}_H \boldsymbol{G}_S \Delta\lambda
\end{aligned}
\tag{16.16}
$$

The plastic increment can now be solved as

$$
\Delta\lambda = \left(\boldsymbol{G}_S^T \boldsymbol{K}_E \boldsymbol{G}_S - \boldsymbol{G}_R^T \boldsymbol{K}_H \boldsymbol{G}_S\right)^{-1} \boldsymbol{G}_S^T \boldsymbol{K}_E \Delta \boldsymbol{v}
\tag{16.17}
$$

Substituting $\Delta\lambda$ the elasto-plastic stiffness of the element now becomes

$$
\begin{aligned}
\Delta \boldsymbol{S} &= \boldsymbol{K}_E \Delta \boldsymbol{v} - \boldsymbol{K}_E \boldsymbol{G}_S \Delta\lambda \\
&= \boldsymbol{K}_E \Delta \boldsymbol{v} - \boldsymbol{K}_E \boldsymbol{G}_S \left(\boldsymbol{G}_S^T \boldsymbol{E}_E \boldsymbol{G}_S - \boldsymbol{G}_R^T \boldsymbol{K}_H \boldsymbol{G}_S\right)^{-1} \boldsymbol{G}_S^T \boldsymbol{K}_E \Delta \boldsymbol{v} \\
&= \left(\boldsymbol{K}_E - \boldsymbol{K}_E \boldsymbol{G}_S \left(\boldsymbol{G}_S^T \boldsymbol{K}_E \boldsymbol{G}_S - \boldsymbol{G}_R^T \boldsymbol{K}_H \boldsymbol{G}_S\right)^{-1} \boldsymbol{G}_S^T \boldsymbol{K}_E\right) \Delta \boldsymbol{v} \\
&= \boldsymbol{k}_T^{ep} \Delta \boldsymbol{v}
\end{aligned}
\tag{16.18}
$$

Hardening functions $R(\mathbf{v}^P)$

In the context of joint modelling, the hardening function $\boldsymbol{R(v^P)}$ and $\boldsymbol{k}^h$ can be directly derived from the nonlinear P-* and M-2 curves (3.19), where N_U and M_U are the maximum joint capacities under axial force and bending, respectively.

$$\frac{N}{N_U} \;'\; 1 \;\&\; A\left(1 \;\&\; (1 \,\%\, \frac{1}{\sqrt{A}})\;\; e^{\& \frac{B\;*}{Q_y\, f_y\; D}} \right)^2$$

$$\frac{M}{M_U} \;'\; 1 \;\&\; A\left(1 \;\&\; (1 \,\%\, \frac{1}{\sqrt{A}})\;\; e^{\& \frac{B\;2}{Q_y\, f_y}} \right)^2 \tag{16.19}$$

The plasticity formulation requires that the joint element is assigned an elastic stiffness, a limit to the elastic range, and a plastic stiffness or hardening function. Thus, the plastic interaction function is given by

$$' \;\; ' \; f\left(\frac{N}{N_0\, R_N},\; \frac{M_{ipb}}{M_{ipb,0}\, R_{ipb}},\; \frac{M_{opb}}{M_{opb,0}\, R_{opb}} \right) \;\&\;\; 1 \;\;'\;\; 0 \tag{16.20}$$

where N_0 and M_0 denotes the limits of the elastic range and R_k are the hardening functions. Comparing (3.19) and (3.20), the hardening functions are given by

$$R_N \;'\; \frac{N_U}{N_0} \left\{ 1 \;\&\; A\left(1 \;\&\; (1 \,\%\, \frac{1}{\sqrt{A}})\;\; e^{\& \frac{B\;*}{Q_y\, f_y\; D}} \right)^2 \right\} \tag{16.21}$$

The elasto-plastic stiffness for the individual degrees of freedom are then given by

$$\frac{1}{k} \;'\; \frac{1}{k_e} \,\%\, \frac{1}{k_h} \tag{16.22}$$

where k_e is the elastic stiffness and k_h is the hardening stiffness. The resulting stiffness k should be equal to the derivatives of the P-* and M-2 curves (3.23).

$$k_N \;'\; \frac{dN}{d\,*} \;'\; \&\,2\, \frac{N_U A}{N_0}\; (1 \,\%\, \frac{1}{\sqrt{A}})\; \frac{B\;\;*}{Q_y\, f_y\; D} \left(1 \;\&\; (1 \,\%\, \frac{1}{\sqrt{A}})\;\; e^{\& \frac{B\;*}{Q_y\, f_y\; D}} \right) e^{\& \frac{B\;*}{Q_y\, f_y\; D}}$$

$$k_M \;'\; \frac{dM}{d\,2} \;'\; \&\,2\, \frac{M_U A}{M_0}\; (1 \,\%\, \frac{1}{\sqrt{A}})\; \frac{B\;\;2}{Q_y\, f_y} \left(1 \;\&\; (1 \,\%\, \frac{1}{\sqrt{A}})\;\; e^{\& \frac{B\;2}{Q_y\, f_y}} \right) e^{\& \frac{B\;2}{Q_y\, f_y}} \tag{16.23}$$

With the elastic stiffness taken as the secant stiffness from the origin to the limit of the “elastic” region, the hardening stiffness can then be directly calculated from (3.22).

Gradients to the yield surface

The change in yield function due to a change in external forces is given by

$$\boldsymbol{g}_{S,i}^{T} = \frac{\partial\Phi}{\partial \boldsymbol{S}_i} = \left[\frac{\partial\Phi}{\partial N},\ \frac{\partial\Phi}{\partial M_{ipb}},\ \frac{\partial\Phi}{\partial M_{opb}} \right]_i \tag{16.24}$$

The change in yield function due to changes in surface shape and extension of the yield surface is given by

$$\boldsymbol{g}_{R,i}^{T} = \frac{\partial\Phi}{\partial \boldsymbol{R}_i} = \left[\frac{\partial\Phi}{\partial R_N},\ \frac{\partial\Phi}{\partial R_{ipb}},\ \frac{\partial\Phi}{\partial R_{opb}} \right]_i \tag{16.25}$$

Index i refers to beam end 1 and beam end 2.

Using the general, user defined, interaction function from (3.1), the instantaneous yield surface (including hardening) is given by

$$\Phi = \left(\frac{N}{N_0 R_N}\right)^{\beta_1} + \left(\left(\frac{M_{ipb}}{M_{ipb,0} R_{ipb}}\right)^{\beta_3} + \left(\frac{M_{opb}}{M_{opb,0} R_{opb}}\right)^{\beta_4} \right)^{\beta_5} - 1 = 0 \tag{16.26}$$

The derivatives are

$$\frac{\partial\Phi}{\partial N} = \frac{\beta_1}{N_0 R_N} \left(\frac{N}{N_0 R_N}\right)^{(\beta_1 - 1)}$$

$$\frac{\partial\Phi}{\partial M_{ipb}} = \frac{\beta_5 \beta_3}{M_{ipb,0} R_{ipb}} \left(\frac{M_{ipb}}{M_{ipb,0} R_{ipb}}\right)^{(\beta_3 - 1)} \left(\left(\frac{M_{ipb}}{M_{ipb,0} R_{ipb}}\right)^{\beta_3} + \left(\frac{M_{opb}}{M_{opb,0} R_{opb}}\right)^{\beta_4} \right)^{(\beta_5 - 1)} \tag{16.27}$$

$$\frac{\partial\Phi}{\partial M_{opb}} = \frac{\beta_5 \beta_4}{M_{opb,0} R_{opb}} \left(\frac{M_{opb}}{M_{opb,0} R_{ipb}}\right)^{(\beta_4 - 1)} \left(\left(\frac{M_{ipb}}{M_{ipb,0} R_{ipb}}\right)^{\beta_3} + \left(\frac{M_{opb}}{M_{opb,0} R_{opb}}\right)^{\beta_4} \right)^{(\beta_5 - 1)}$$

$$\frac{\partial\Phi}{\partial R_N} = -\beta_1 \frac{N}{N_0} \left(\frac{1}{R_N}\right)^2 \left(\frac{N}{N_0 R_N}\right)^{(\beta_1 - 1)}$$

$$\begin{aligned} \frac{\partial\Phi}{\partial R_{ipb}} &= -\beta_5 \beta_3 \frac{M_{ipb}}{M_{ipb,0}} \left(\frac{1}{R_{ipb}}\right)^2 \left(\frac{M_{ipb}}{M_{ipb,0} R_{ipb}}\right)^{(\beta_3 - 1)} \\ &\quad \cdot \left(\left(\frac{M_{ipb}}{M_{ipb,0} R_{ipb}}\right)^{\beta_3} + \left(\frac{M_{opb}}{M_{opb,0} R_{opb}}\right)^{\beta_4} \right)^{(\beta_5 - 1)} \\ \frac{\partial\Phi}{\partial R_{opb}} &= -\beta_5 \beta_4 \frac{M_{opb}}{M_{opb,0}} \left(\frac{1}{R_{opb}}\right)^2 \left(\frac{M_{opb}}{M_{opb,0} R_{opb}}\right)^{(\beta_4 - 1)} \\ &\quad \cdot \left(\left(\frac{M_{ipb}}{M_{ipb,0} R_{ipb}}\right)^{\beta_3} + \left(\frac{M_{opb}}{M_{opb,0} R_{opb}}\right)^{\beta_4} \right)^{(\beta_5 - 1)} \end{aligned} \tag{16.28}$$

Integration of constituitive equations

The global load increment determines the total deformations for each element in the structure. To speed up the analysis procedure, a simple cutting plane algorithm is introduced to integrate the constituitive equations, i.e. to determine the distribution between elastic and plastic displacement for each member (Ortiz and Simo, 1986).

An initial trial step is executed for each member, assuming that the element remains elastic for the full increment in displacements

$$
\begin{aligned}
\Delta \boldsymbol{S}^{\boldsymbol{E}}_{\boldsymbol{n+1}} &= \boldsymbol{k}^{E}_{T}\, \Delta \boldsymbol{v}^{\boldsymbol{tot}} \\
\boldsymbol{S}_{\boldsymbol{n+1}} &= \boldsymbol{S}_{n} + \Delta \boldsymbol{S}^{E}_{n+1}
\end{aligned}
\tag{16.29}
$$

Subscript *n* refers to the previous, converged load step and subscript *n+1* refers to the current step.

The resulting internal forces are then checked against the current yield condition to see if the assumption holds. If the yield condition is violated, i.e. $\Phi(\boldsymbol{S}_{\boldsymbol{n+1}}) > 0$, some of the element deformations will have to be taken as plastic deformations. The plastic deformations are determined by an iterative procedure, repeated until

a. the state of forces satisfy the yield criterion $\Phi(\boldsymbol{S}_{\boldsymbol{n+1}}) = 0$

b. the plastic deformations satisfy the hardening rule

c. the elastic and plastic deformations equals the total incremental deformations

This is expressed by the following steps:

1. The plastic increment is calculated from the consistency condition. The consistency condition during local force iterations is formulated as

$$
\begin{aligned}
\Phi^{k+1}_{n+1} &= \Phi^{k}_{n+1} + d\Phi \\
&= \Phi^{k}_{n+1} + \frac{\partial \Phi}{\partial \boldsymbol{S}}\, d\boldsymbol{S}^{k}_{n+1} + \frac{\partial \Phi}{\partial \boldsymbol{R}}\, d\boldsymbol{R}^{k}_{n+1} \\
&= \Phi^{k}_{n+1} + (\boldsymbol{g}^{k}_{S,\,n+1})^{T}\, d\boldsymbol{S}^{k}_{n+1} + (\boldsymbol{g}^{k}_{R,\,n+1})^{T}\, \boldsymbol{k}^{h}_{R,\,n+1}\, \boldsymbol{g}^{k}_{S,\,n+1}\, d\Delta\lambda \\
&= 0
\end{aligned}
\tag{16.30}
$$

   where prefix *d()* denotes iterative changes in $\boldsymbol{S}$, $\boldsymbol{R}$ and $\Delta\lambda$. External, total deformations remains fixed during these iterations. Thus, $d\boldsymbol{v}^{tot} = \boldsymbol{0}$ and

$$
\begin{aligned}
d\boldsymbol{S} &= \boldsymbol{k}_{E}\,(d\boldsymbol{v}^{tot} - d\boldsymbol{v}^{P}) \\
&= -\boldsymbol{k}_{E}\, d\boldsymbol{v}^{P} \\
&= -\boldsymbol{k}_{E}\, \boldsymbol{g}\, d\Delta\lambda
\end{aligned}
\tag{16.32}
$$

The consistency condition can now be re-written as

$$
\begin{aligned}
\Phi^{k+1}_{n+1} &= \Phi^{k}_{n+1} + (\boldsymbol{g}^{k}_{S,\,n+1})^{T}\, d\boldsymbol{S}^{k}_{n+1} + (\boldsymbol{g}^{k}_{R,\,n+1})^{T}\, \boldsymbol{H}^{k}_{R,\,n+1}\, \boldsymbol{g}^{k}_{S,\,n+1}\, d\Delta\lambda \\
&= \Phi^{k}_{n+1} - (\boldsymbol{g}^{k}_{S,\,n+1})^{T}\, \boldsymbol{k}^{E}_{n+1}\, \boldsymbol{g}^{k}_{S,\,n+1}\, d\Delta\lambda + (\boldsymbol{g}^{k}_{R,\,n+1})^{T}\, \boldsymbol{k}^{h}_{R,\,n+1}\, \boldsymbol{g}^{k}_{S,\,n+1}\, d\Delta\lambda \\
&= 0
\end{aligned}
\tag{16.33}
$$

and $d\Delta\lambda$ can be solved as

$$
d\Delta\lambda = \left(-(\boldsymbol{g}^{k}_{S,\,n+1})^{T}\boldsymbol{k}^{E}\boldsymbol{g}^{k}_{S,\,n+1} + (\boldsymbol{g}^{k}_{R,\,n+1})^{T}\, \boldsymbol{k}^{h}_{R,\,n+1}\, \boldsymbol{g}^{k}_{S,\,n+1}\right)^{-1} \Phi^{k}_{n+1} \tag{16.34}
$$

2. The accumulated plastic increment is calculated

$$
\Delta\lambda^{k+1}_{n+1} = \Delta\lambda^{k}_{n+1} + d\Delta\lambda \tag{16.35}
$$

3. Accumulated plastic deformations are calculated

$$
\Delta\boldsymbol{v}^{P\,(k+1)}_{n+1} = \Delta\boldsymbol{v}^{P\,(k)}_{n+1} + \boldsymbol{g}^{k}_{n+1}\, d\Delta\lambda \tag{16.36}
$$

4. Internal forces are calculated as the accumulated forces up to the last increment, minus the relaxation in forces due to plasticity

$$
\begin{aligned}
\boldsymbol{S}^{k+1}_{n+1} &= \boldsymbol{S}^{0}_{n+1} - \boldsymbol{k}^{E}\, \Delta\boldsymbol{v}^{P,\,(k+1)}_{n+1} \\
&= \boldsymbol{S}^{k}_{n+1} - \boldsymbol{k}^{E}\boldsymbol{g}^{k}_{n+1}\, d\Delta\lambda
\end{aligned}
\tag{16.37}
$$

5. The hardening corresponding to the plastic deformations are calculated as

$$
\boldsymbol{R}^{k+1}_{n+1} = \boldsymbol{R}^{k}_{n+1} + \boldsymbol{k}^{h}_{R,\,n+1}\, \boldsymbol{g}^{k}_{S,\,n+1}\, d\Delta\lambda \tag{16.38}
$$

6. The internal forces are checked against to see if the current yield condition is satisfied

$$
\Phi(\boldsymbol{S}_{n+1}) \leq 0 \tag{16.39}
$$

Repeat from step 1 until the yield condition is satisfied.

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