3 THEORY

3.1 INTRODUCTION

The computer code FAHTS, (Fire And Heat Transfer Simulations), has been developed in connection with the SINTEF Project:

"Integrated Analysis of Steel and Aluminium Structures Exposed to Fire"

FAHTS is designed to form a 'link' between the fire simulations, (KAMELEON/FIRINT) and the mechanical response simulations, (USFOS), but has several simplified fire scenario models available in addition.

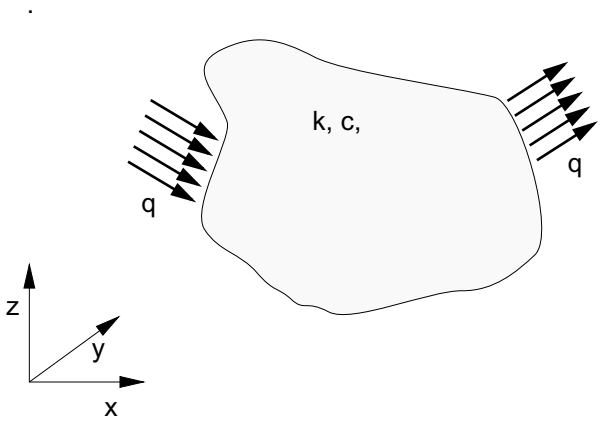
The structural description file used by USFOS is also used by FAHTS and this implies that the beam element FEM-model has to be re-meshed automatically to obtain a 3-D representation of the structure. The results from the temperature simulations are transferred back to beam element temperature load data required by USFOS.

FAHTS is a Finite Element program for transient heat transfer simulations using an implicit time integration scheme to perform the incremental solution.

FAHTS is based on a general formulation but is special designed to be an effective tool used in connection with USFOS.

3.2 FUNDAMENTAL THEORY OF THERMAL ENERGY BALANCE

3.2.1 Constitutive equations and definitions for heat flow calculations



Fourier's law for heat flow :

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + q \qquad (3.2.1)$$

Where:

ρ	:	Mass density	[kg/m^3]
c	:	Heat capacity	[J/kgK]
T	:	Temperature	[K]
t	:	Time	[sec]
$k_{x,y,z}$	:	Thermal conductivity in x, y and z direction	[W/mK]
x,y,z	:	Spatial coordinates	[m]
q	:	Heat input to the system	[W/m^3]

3.2.2 Spatial discretization

The Finite Element approach:

For the general case it is impossible to obtain a analytical solution for the transient heat transfer problem, and in this section the transformation of the differential equation describing the continuous problem to a discrete model applicable for an efficient numerical solution approach is discussed.

For the 2-dimensional case the differential equation, 3.2.1 yields:

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}(k_x\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k_y\frac{\partial T}{\partial y}) + q \qquad (3.2.2)$$

The term **q** denoted the heat generation contribution will in fact fro the 2-dimensional case include the terms connected to heat exchange over the intersurface between material and surroundings.

It should be noted that this term does not represent a true boundary condition in mathematical terms, and is here q = q_s / t where $\mathbf{q_s}$ is the intermedium cross heat flow, (f.inst. netto radiant heat flux), and **t** is the thickness of the continuum.

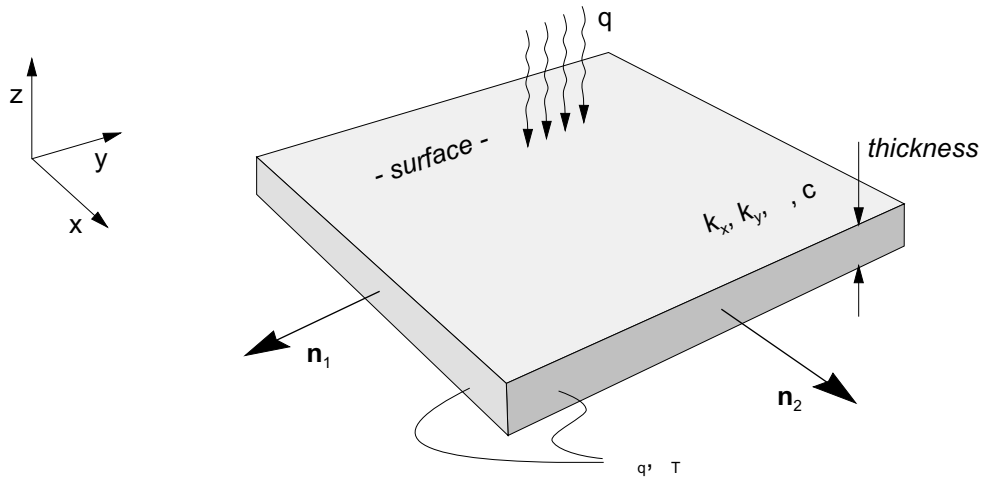


Figure 3.2.2.1 Definition of symbols

The boundary conditions inherent for this type of problem may be written as:

$$T - \bar{T} = 0 \qquad \text{on } \Gamma_T$$

$$k_x\frac{\partial T}{\partial n_x} + k_y\frac{\partial T}{\partial n_y} - \bar{q} = 0 \qquad \text{on } \Gamma_q \qquad (3.2.3)$$

where $\bar{T}$ and $\bar{q}$ denotes prescribed values of temperature and heat flow across the (true) boundaries of the continuous system, and $\mathbf{n_x}$ **and** $\mathbf{n_y}$ are the direction normal to the **boundary** $\mathbf{\Gamma}$ with prescribed conditions.

The above differential equations may for convenience be written on the following general form:

$$\boldsymbol{A(u)} = \boldsymbol{Lu} + \boldsymbol{p} = 0 \qquad \text{in domain } \Omega$$

$$\boldsymbol{B(u)} = \mathbf{M} + \boldsymbol{t} = 0 \qquad \text{in domain } \Gamma \tag{ 3.2.4 }$$

Here the linear operators **L** and **M** are introduced to indicate that the formulation in question is linear in the unknown **u** ≡ T.

In general two distinct procedures are available for obtaining discrete approximations to the above continuous problem description. The first one, which will employed in the present context, is the method of weighted residuals, (or better known as the Galerkin method). The second method is the formation of variational functionals for which the conditions for stationarity is considered.

In the following the weighted residual method will be applied for the transient heat conduction problem in order to arrive to a system of ordinary differential equations from which the unknown temperature T may be calculated.
It is here convenient to perform what is termed a partial discretization which means that the real domain of independent variables Ω(x,y,t) is reduced to a sub-domain $\bar{\Omega}$(x,y) leaving out the time variable t.
As a next step, s suitable time stepping algorithm may be applied to the discrete system of equations in order to calculate the time-dependent solution.

As the differential equation(s) and boundary conditions according to (3.2.3) have to be zero at each point in the domain $\bar{\Omega}$(x,y), it follows that:

$$\int \bar{v} A(T)\, d\bar{\Omega} + \int \bar{\bar{v}} B(T)\, d\Gamma = 0 \tag{ 3.2.5 }$$

where v and $\bar{v}$ are arbitrary functions.
Here A(t) and B(T) represent one differential equation and the corresponding boundary condition. The proof for this statement is obvious as it is easily seen that if A(T)#0 at any position in the domain, a function v may be found which makes the integral in (3.2.5) different from zero.

Next, the continuous unknown temperature T defined in the domain Ω(x,y,t) is approximated by the expansion:

$$T(x,y,t) = \sum_{i=1}^{n} N_i(x,y)\, T_i(t) = \boldsymbol{NT} \tag{ 3.2.6 }$$

where N_i(x,y) denote the n shape functions prescribed in terms of the independent variables x and y defining the sub-domain $\bar{\Omega}$, and T_i the unknown time dependent parameters, (in FEM terms the generalized coordinates).

Obviously, the spatial approximation made in (3.2.6) cannot satisfy the differential equation. An approximation to the integral form in Eq (3.2.5) by instead of the arbitrary function **v**, we introduce a finite set of prescribed functions:

$v = w_j$, $\bar{v} = \bar{w}_j$, j = 1 , n

which gives a set of ordinary differential equations:

$$\int_{\bar{\Omega}} w_j A(\boldsymbol{NT})\, d\Omega + \int_{\Gamma} \bar{w}_j B(\boldsymbol{NT})\, d\Gamma = 0 \quad , \quad j = 1, n \tag{ 3.2.7 }$$

In Eq (3.2.7) A(**N T**) represents a residual resulting from the substitution of the approximation in Eq (3.2.6) into the differential equation, and B(**N T**) a residual on the boundary conditions. Eq (3.2.7) thus represents a **weighted integral** of these residuals. By choosing the weight function w_j' equal to the shape functions N_j the Galerkin method is obtained / /.
This method will generally lead to symmetric equation system matrices which is a preferable property in the numerical solution process.

Furthermore, Eq (3.2.7) ensures that the error (or residual) introduced by the spatial discretization averaged over the integration domain is zero, or is correct in terms of energy.

The selection of suitable shape functions will be introduced later, but at this stage it is interesting to note that these functions may be chosen on the **element level** and integration over the total domain Ω is obtained by **summation over all elements, ie the finite element method**.

Alternatively, "global" shape functions may be chosen if desired.

For the transient heat transfer problem we have:

$$A(T) = \frac{\partial}{\partial x}(k_x\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k_y\frac{\partial T}{\partial y}) + Q - c\frac{\partial T}{\partial t} = 0$$

$$B(T) = T - \overline{T} = 0 \qquad \text{on } \Gamma_T \qquad (3.2.8)$$

$$k_x\frac{\partial T}{\partial n_x} + k_y\frac{\partial T}{\partial n_y} - \overline{q} = 0 \qquad \text{on } \Gamma_q$$

On basis of Eq (3.2.7) we get:

$$\int_{\Omega} w_j[\frac{\partial}{\partial x}(k_x\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k_y\frac{\partial T}{\partial y})]dA + \int_{\Omega} w_j[Q - c\frac{\partial T}{\partial t}]dA + \int_{\Gamma_q}\overline{w_j}[k\frac{\partial T}{\partial n} - \overline{q}]d\Gamma = 0 \qquad (3.2.9)$$

The first integral in (3.2.9) is integrated by parts by use of **Greens theorem**:

$$\int_{\Omega} w_j\frac{\partial}{\partial x}(k_x\frac{\partial T}{\partial x})dx\,dy = -\int_{\Omega}\frac{\partial w_j}{\partial x}(k_x\frac{\partial T}{\partial x})dx\,dy + \oint_{\Gamma} w_j(k_x\frac{\partial T}{\partial x})n_x d\Gamma \qquad (3.2.10)$$

$$\int_{\Omega} w_j\frac{\partial}{\partial y}(k_y\frac{\partial T}{\partial y})dx\,dy = -\int_{\Omega}\frac{\partial w_j}{\partial y}(k_y\frac{\partial T}{\partial y})dx\,dy + \oint_{\Gamma} w_j(k_y\frac{\partial T}{\partial y})n_y d\Gamma$$

Thus, re-writing Eq (3.2.9) we get:

$$- \int_{\bar{\Omega}} w_j [\frac{\partial w_j}{\partial x}(k_x \frac{\partial T}{\partial x}) + \frac{\partial w_j}{\partial y}(k_y \frac{\partial T}{\partial y}) - w_j(Q - c\frac{\partial T}{\partial t})]\, dx\, dy +$$

$$\oint_{\Gamma} w_j (k_x \frac{\partial T}{\partial x} n_x + k_y \frac{\partial T}{\partial y} n_y) d\Gamma + \int_{\Gamma_q} \overline{w_j}(k_x \frac{\partial T}{\partial x} n_x + k_y \frac{\partial T}{\partial y} n_y - \bar{q}) d\Gamma = 0 \qquad (3.2.11)$$

By putting $\mathbf{w_j = -\bar{w}_j}$ (as the weight functions originally could be chosen arbitrarily and making use of $\mathbf{\Gamma = \Gamma_q + \Gamma_T}$, we get:

$$\int_{\bar{\Omega}} w_j [\frac{\partial w_j}{\partial x}(k_x \frac{\partial T}{\partial x}) + \frac{\partial w_j}{\partial y}(k_y \frac{\partial T}{\partial y})] d\bar{\Omega} + \int_{\bar{\Omega}} w_j c\rho \frac{\partial T}{\partial t} d\bar{\Omega} - \int_{\bar{\Omega}} w_j Q d\bar{\Omega}$$

$$- \int_{\Gamma_q} w_j \bar{q}\, d\Gamma - \int_{\Gamma_T} w_j (k_x \frac{\partial T}{\partial x} n_x + k_y \frac{\partial T}{\partial y} n_y) d\Gamma = 0 \qquad j = 1\ ,\ n \qquad (3.2.12)$$

It is observed from Eq (3.2.12) that the natural boundary condition

$B(T) = k_x \frac{\partial T}{\partial n_x} + k_y \frac{\partial T}{\partial n_y} - \bar{q} = 0$ on Γ_q is automatically take care of.

Furthermore, the last term in Eq (3.2.12) disappear if we select $\mathbf{w_j = 0}$ **on the boundary** $\mathbf{\Gamma_T}$.

When introducing the weight functions according to Galerkin, $w_j = N_j$ and also invoking Eq (3.2.6) we obtain:

$$\int_{\bar{\Omega}} [\frac{\partial N_j}{\partial x} k_x \frac{\partial N_i}{\partial x} + \frac{\partial N_j}{\partial y} k_y \frac{\partial N_i}{\partial y}] d\bar{\Omega} \cdot T_i(t) + \int_{\bar{\Omega}} N_j c\rho N_i d\bar{\Omega} \cdot \frac{\partial T_i(t)}{\partial t} = \int_{\bar{\Omega}} N_j Q d\bar{\Omega} + \int_{\Gamma_q} N_j \bar{q}\, dl$$

(3.2.13)

If the integration in Eq (3.2.13) is performed over a finite element, we re-write the heat flow balance on the following form:

$$\boldsymbol{m\dot{T} + kT = Q_g + Q_q = Q} \qquad (3.2.14)$$

where

$$\boldsymbol{k} = \int_{A_e} [\frac{\partial N_j}{\partial x} k_x \frac{\partial N_i}{\partial x} + \frac{\partial N_j}{\partial y} k_y \frac{\partial N_i}{\partial y}] dA = \int_{A_e} \boldsymbol{N}_{,x}^T \boldsymbol{k} \boldsymbol{N}_{,x} dA \quad \text{(element conductivity matrix)}$$

$$\boldsymbol{m} = \int_{A_e} \rho c N_j N_i dA = \int_{A_e} \rho c \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{N} dA \quad \text{(element specific heat matrix)}$$

$$\boldsymbol{Q_g} = \int_{A_e} N_j Q dA = \int_{A_e} \boldsymbol{N} Q dA \quad \text{(element heat flow vector, surface)}$$

$$\boldsymbol{Q_q} = \int_{L_B} N_j \bar{q} dl = \int_{L_B} \boldsymbol{N} \bar{q} dl \quad \text{(element heat flow vector, edges)}$$

(3.2.15)

In Eq (3.2.15) the matrix notation for the element matrices and load vectors are introduced.

For the case when the boundary heat flow denoted $\bar{q}$ is governed by temperature gradients, ie:

$$\bar{q} = \overline{q_c} + \alpha(T_{bound} - T_s) \qquad (3.2.16)$$

where

$\bar{q}_c$: Constant heat flow term
α : "total" heat flow coefficient
T_{bound} : Element boundary temperature
T_s : Temperature of surrounding objects exchanging heat with the element

which inserted into the equation for $\mathbf{Q}_q$ in Eq (3.2.15) gives:

$$\int_{\Gamma_q} N_j \bar{q} dl = \int_{\Gamma_q} N_j [\ \overline{q_c} + \alpha(T_{bound} - T_s)\ ] dl =$$
$$\int_{\Gamma_q} N_j (\overline{q_c} - \alpha T_s) dl + \int_{\Gamma_q} N_j \alpha T_{bound} dl \qquad (3.2.17)$$

The first term in Eq (3.2.17) represents a load term which can be directly added to the local vector **Q**. However, the second term contains the unknown temperature and represents a contribution to **m** which may be written on the form:

$$\boldsymbol{m_b} = \alpha \int_{\Gamma_q} \boldsymbol{N_B}^{\boldsymbol{T}} \boldsymbol{N_B} dl$$

where $\mathbf{N}_B$ denotes the boundary shape function which is obtained from **N** with suitable boundary coordinates inserted, (ξ or η).

The Fourier equation for the entire system is then expressed on matrix form as follows:

$$\boldsymbol{M_i \dot{T}_i + K_i T_i = Q_i} \qquad (3.2.18)$$

Where:

- $\mathbf{K_i}$: Thermal Conductivity Matrix at time **i**
- $\mathbf{T_i}$: Nodal Temperature Vector at time **i**
- $\mathbf{M_i}$: Mass Matrix including heat capacity at time **i**
- $\mathbf{\dot{T}_i}$: Nodal Temperature Rate Vector at time **i**
- $\mathbf{Q_i}$: Nodal Consistent Heat Vector at time **i**

3.2.3 Time Domain Integration

In section 3.2.2 the transient temperature field problem was discretized in the space domain resulting in a linear set of ordinary different equations.

STATIONARY SOLUTION

The special case, **stationary solution** is solved as follows:

The inertia term is removed from the equation:

$$\boldsymbol{K}\boldsymbol{T} = \boldsymbol{Q}$$

with solution:

$$\boldsymbol{T} = \boldsymbol{K}^{-1}\boldsymbol{Q}$$

K : Thermal Conductivity Matrix, (must be temperature independent)
T : Nodal Temperature Vector, (the solution)
Q : Nodal Consistent Heat Vector.

TRANSIENT SOLUTION

For the heat transfer problem non-linearities have to be accounted for as both the conductance and heat capacity in general are temperature dependent in addition to the heat load vector which is strongly dependent on the temperature.

The problem may for this case be written on the form:

$$\boldsymbol{M}(T)\dot{\boldsymbol{T}} + \boldsymbol{K}(T)\boldsymbol{T} = \boldsymbol{Q}(T) \qquad (3.2.19)$$

In this section the numerical integration of Eq. (3.2.19) in the time domain is described.
The solution will be formulated on a recursive incremental form to find an approximation of the temperature state T_i at time t_i given the solution T_{i-1} at time t_{i-1} assuming a time interval $\Delta t = t_i - t_{i-1}$.

In the time interval Δt the temperature **T** can be taken to vary as a polynomial, and for the present case it is assumed that sufficiently accuracy is obtained by selecting the linear expansion given by:

$$\boldsymbol{T} \approx \boldsymbol{T}(\tau) = \boldsymbol{T}_{i-1} + \frac{\tau}{\Delta t}(\boldsymbol{T}_i - \boldsymbol{T}_{i-1}) \qquad (3.2.20)$$

where $\tau = t - t_{i-1}$.

This approximation is illustrated in Figure 3.2.3.1.

Employing the weighted residual approach in the time domain Eq. (3.2.19) gives:

$$\int_0^{\Delta t} w(\boldsymbol{M}\dot{\boldsymbol{T}} + \boldsymbol{K}\boldsymbol{T} - \boldsymbol{Q})\,d\tau = 0 \qquad (3.2.21)$$

where w represents a yet undetermined weight function.

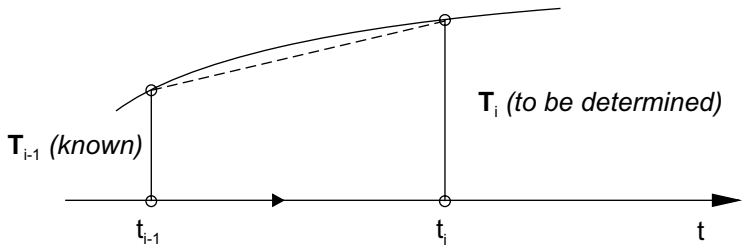


Figure 3.2.3.1 Approximation to **T** in the time domain

Introducing a weight parameter θ given by /Zienkiewicz/ :

$$\theta = \frac{1}{\Delta t} \frac{\int_0^{\Delta t} w\tau\, d\tau}{\int_0^{\Delta t} w\, d\tau} \qquad (3.2.22)$$

eq (3.2.21) can be written as:

$$\int_0^{\Delta t} w\boldsymbol{M}(\boldsymbol{T}_i - \boldsymbol{T}_{i-1})\frac{1}{\Delta t} d\tau + \int_0^{\Delta t} w\boldsymbol{K}[\ \boldsymbol{T}_{i-1} + \frac{\tau}{\Delta t}(\boldsymbol{T}_i - \boldsymbol{T}_{i-1})\ ] d\tau = \int_0^{\Delta t} w\boldsymbol{Q}\, d\tau \qquad (3.2.23)$$

Assuming **M** and **K** constant in the small time interval:

$$\boldsymbol{M}(\boldsymbol{T}_i - \boldsymbol{T}_{i-1})\frac{1}{\Delta t} + \boldsymbol{K}[\ \boldsymbol{T}_{i-1} + \theta(\boldsymbol{T}_i - \boldsymbol{T}_{i-1})\ ] = \overline{\boldsymbol{Q}} \qquad (3.2.24)$$

where $\overline{\mathbf{Q}}$ represents an average of the equivalent load vector. If the heat flux 'load' **Q** is assumed to vary as a linear function over the time interval Δt the averaged heat flux becomes:

$$\overline{\boldsymbol{Q}} = \boldsymbol{Q}_{i-1} + \theta(\boldsymbol{Q}_i - \boldsymbol{Q}_{i-1}) \qquad (3.2.25)$$

For the nonlinear problem Eq (3.2.24) may be re-written to the incremental form:

$$\boldsymbol{M}_i\frac{\Delta \boldsymbol{T}}{\Delta t} + \boldsymbol{K}_i\boldsymbol{T}_{i-1} + \theta\boldsymbol{K}_i\Delta\boldsymbol{T} = \overline{\boldsymbol{Q}} \qquad (3.2.26)$$

$$(\boldsymbol{K}_i + \frac{1}{\theta\Delta t}\boldsymbol{M}_i)\Delta T = \frac{1}{\theta}\overline{\boldsymbol{Q}} - \frac{1}{\theta}\boldsymbol{K}_i\boldsymbol{T}_{i-1} \qquad (3.2.27)$$

$$(\boldsymbol{K}_i + \frac{1}{\theta\Delta t}\boldsymbol{M}_i)\Delta T = \boldsymbol{Q}_i + \frac{1-\theta}{\theta}\boldsymbol{Q}_{i-1} - \frac{1}{\theta}\boldsymbol{K}_i\boldsymbol{T}_{i-1} \qquad (3.2.28)$$

In the present FAHTS implementation the weight parameter is chosen to θ **= 1/2** corresponding pure averaging over the time interval Δt. This approach is known as the Crank - Nicolson or trapezoidal rule /Zienkiewicz/.

If the term $K_i\ T_{i-1}$ is replaced by $K_{i-1}\ T_{i-1}$ the following incremental form is obtained:

$$(\boldsymbol{K}_i + \frac{2}{\Delta t}\boldsymbol{M}_i)\Delta T = \boldsymbol{Q}_i + \boldsymbol{Q}_{i-1} - 2\boldsymbol{K}_{i-1}\boldsymbol{T}_{i-1} \qquad (3.2.29)$$

This type of equation is usually solved by employing some sort of iterative scheme, f.ins the well known Newton - Raphson procedure.
Here a procedure with incrementation and equilibrium correction is employed.
Hence the term $\mathbf{Q}_{i-1}$ is substituted by:

$$\boldsymbol{Q}_{i-1} = \boldsymbol{M}_{i-1}\dot{\boldsymbol{T}}_{i-1} + \boldsymbol{K}_{i-1}T_{i-1}$$

giving

$$(\boldsymbol{K}_i + \frac{2}{\Delta t}\boldsymbol{M}_i)\Delta T = \boldsymbol{Q}_i + \boldsymbol{M}_{i-1}\dot{\boldsymbol{T}}_{i-1} - \boldsymbol{K}_{i-1}\boldsymbol{T}_{i-1} \qquad (3.2.30)$$

which may be written on the form:

$$\boldsymbol{A}\ \Delta\boldsymbol{T}_i = \boldsymbol{B}$$

with solution:

$$\Delta\boldsymbol{T}_i = \boldsymbol{A}^{-1}\boldsymbol{B}$$

where $$\boldsymbol{A} = (\boldsymbol{K}_i + \frac{2}{\Delta t}\boldsymbol{M}_i)$$

$$\boldsymbol{B} = \boldsymbol{Q}_i - \boldsymbol{K}_{i-1}\boldsymbol{T}_{i-1} + \boldsymbol{M}_{i-1}\dot{\boldsymbol{T}}_{i-1}$$

The unknown temperature solution at time 'i':

$$\boldsymbol{T}_i = \boldsymbol{T}_{i-1} + \Delta\boldsymbol{T}_i$$

Comments:

- It is assumed that the nonlinearity arising from temperature dependent c and k are moderate. For the problems when phase changes are involved, it is recommended to include control of correct enthalpy calculation.
- The heat flux load $\mathbf{Q}(\tau)$ is assumed to vary slowly as a function of τ such that a linear representation over the time interval Δt is realistic.
- The conditions to be satisfied at time t = 0 can be found from Eq (3.2.19):

$$\dot{\boldsymbol{T}}_0 = \boldsymbol{M}_0^{-1}(\boldsymbol{Q}_0 - \boldsymbol{K}_0\boldsymbol{T}_0)$$

- The Crank - Nicolson algorithm is unconditionally stable and the order of accuracy is better than two.
- By selecting the weight parameter θ to other values, other well known integration schemes are obtained, ($\theta = 0$: Explicit Euler, $\theta = 2/3$: Galerkin, $\theta = 1$: Backward Difference).

3.2.4 Surface radiant energy balance

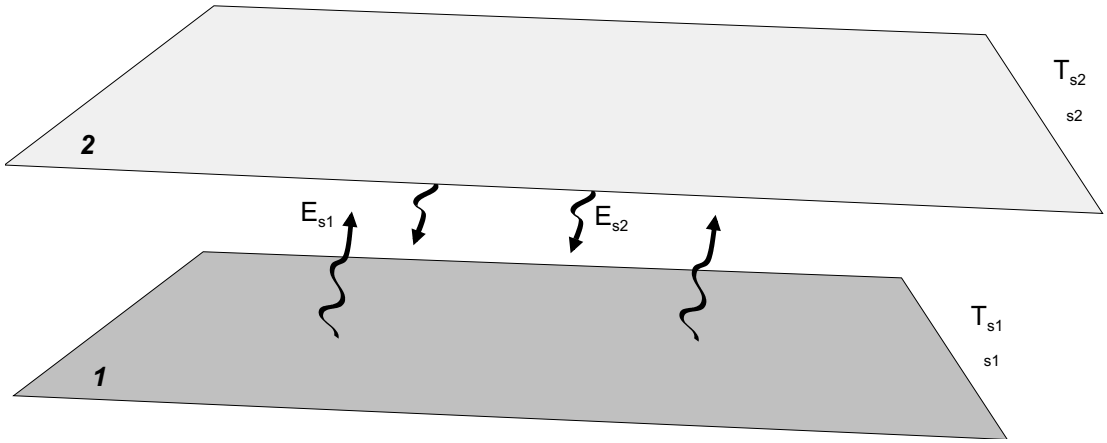


Figure 3.2.1 Radiation between Surfaces

The two surfaces presented in figure 3.2.1 have the following characteristics:

$\mathbf{E_{s1}}$	:	Emitted heat flux from surface 1
$\mathbf{T_{s1}}$	:	Temperature of surface 1
$\boldsymbol{\epsilon_{s1}}$	:	Emissivity coefficient of surface 1
$\mathbf{E_{s2}}$	:	Emitted heat flux from surface 2
$\mathbf{T_{s2}}$	:	Temperature of surface 2
$\boldsymbol{\epsilon_{s2}}$	:	Emissivity coefficient of surface 2

Emitted energy from surface 1:

$$E_{e1} = \sigma \epsilon_{s1} T_{s1}^4$$

Where:

$\boldsymbol{\sigma}$: Stefan-Boltzmann's constant (5.67E-8 W/m^2K^4, SI units!)

Emitted energy from surface 2:

$$E_{e2} = \sigma \epsilon_{s2} T_{s2}^4$$

Surface 1 **absorbs** a fraction of the surface 2 emittance:

$$E_{sa} = \alpha_{s1} E_{s2}$$

Where:

$\mathbf{E_{sa}}$: Absorbed energy at surface 1
$\boldsymbol{\alpha_{s1}}$: Absorption coefficient of surface 1

The netto radiation flux from surface 1 then yields:

$$E_s = \sigma(\epsilon_{s1} T_{s1}^4 - \alpha_{s1} \epsilon_{s2} T_{s2}^4)$$

Where:

$\mathbf{E_s}$	: Netto heat flux from surface 1	[W/m^2	]
$\mathbf{T_{s1}}$	: Temperature of surface 1	[K	]
$\boldsymbol{\epsilon_{s1}}$	: Emissivity coeff. of surface 1	[non-dim	]
$\mathbf{T_{s2}}$	: Temperature of surface 2	[K	]
$\boldsymbol{\epsilon_{s2}}$	: Emissivity coefficient of surface 2	[non-dim	]
$\boldsymbol{\alpha_{s1}}$	: Absorption coefficient surface 2	[non-dim	]

Assuming '**grey-body**' conditions, $\alpha_i = \epsilon_i$, the netto radiation flux from surface 1 then yields:

$$E_s = \sigma \epsilon_{s1}(T_{s1}^4 - \epsilon_{s2} T_{s2}^4)$$

Assuming surface 2 being a '**black-body**', $\epsilon_{s2} = \mathbf{1.0}$, the following special condition is obtained:

$$E_s = \sigma \epsilon_{s1}(T_{s1}^4 - T_{s2}^4)$$

If surface 2 represents only a fraction of the 'hemisphere' observed from surface 1, the netto radiation flux yields:

$$[T_{s1}^4 -$$

Where:

$\mathbf{F_{12}}$: **View factor** of surface 2 [non-dim]

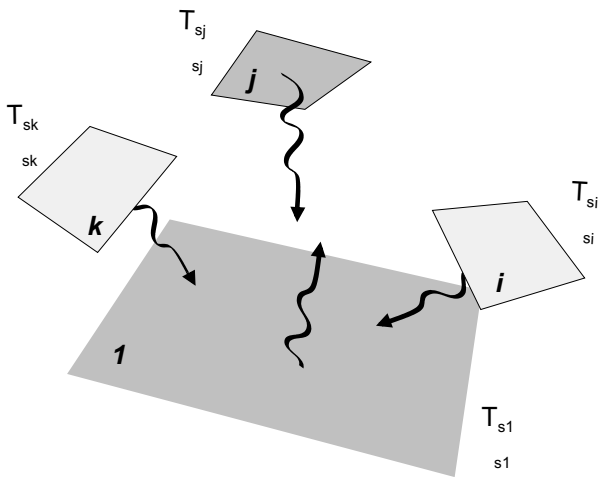


Figure 3.2.2 Radiation exchange between 'n' surfaces

Assuming **'n'** number of surfaces 'visible' from surface 1 as described in figure 3.2.2. The netto radiation then becomes:

$$E_s = \epsilon_{s1}\sigma\left(T_{s1}^4 - \sum_{i=1}^{n} F_{1i}\epsilon_{si}T_{si}^4 \right)$$

Where:

- $\mathbf{F_{1i}}$: **View factor** of surface **'i'** observed from surface 1
- $\mathbf{T_{s1}}$: Temperature of surface 1
- $\boldsymbol{\epsilon_{s1}}$: Emissivity coefficient of surface 1
- $\mathbf{T_{si}}$: Temperature of surface 'i'
- $\boldsymbol{\epsilon_{si}}$: Emissivity coefficient of surface 'i'

A **negative** radiation flux $\mathbf{E_s}$ means **netto absorption**.

3.2.5 Surface convection energy balance

The **convection** energy exchange between a surface and the surrounding fluid is calculated as follows:

$$E_c = C\ (T_s - T_g)$$

Where:

$\mathbf{E_c}$	:	Heat flux from surface to gas	[W/m^2]
C	:	Convection Number	[W/m^2K]
$\mathbf{T_s}$	:	Temperature of surface	[K]
$\mathbf{T_g}$	:	Temperature of surrounding gas	[K]

The **convection number** is dependent on the following parameters:

- Velocity of the surrounding gas
- Thermal conductivity for the gas
- Reynolds number at the current gas temperature

3.3 BASIC ELEMENTS

3.3.1 Quadrilateral heat transfer element

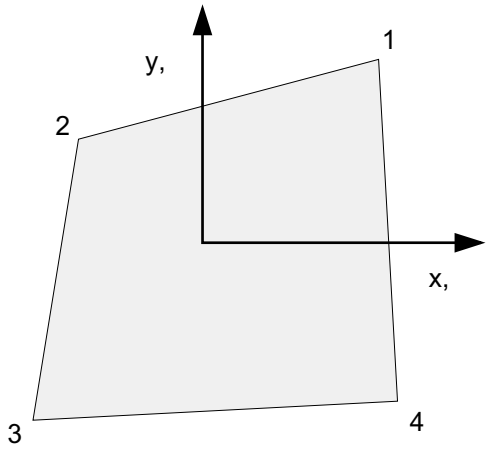


Figure 3.2.3 Isoparametric quadrilateral heat transfer element

The heat flow equation on element level:

$$\boldsymbol{m}\dot{\boldsymbol{T}}_i + \boldsymbol{k}\boldsymbol{T}_i = \boldsymbol{q}_i$$

The element **conductivity matrix** is defined as follows when $k_x = k_y = k$:

$$\boldsymbol{k} = k\int_V (\boldsymbol{N}_{,x}^{\boldsymbol{T}}\boldsymbol{N}_{,x} + \boldsymbol{N}_{,y}^{\boldsymbol{T}}\boldsymbol{N}_{,y})\,dV$$

where $\boldsymbol{N}_{,x}$ denotes $\frac{\partial}{\partial x}\boldsymbol{N}$. V is the element **volume**.

The element **mass matrix** including heat capacity :

$$\boldsymbol{m} = \rho c\int_V (\boldsymbol{N}^{\boldsymbol{T}}\boldsymbol{N})\,dV$$

The consistent **heat source vector**:

$$\boldsymbol{q} = q\int_V (\boldsymbol{N}^T)\,dV$$

Where

ρ	:	Mass density	[kg/m^3]
c	:	Heat capacity	[J/kgK]
k	:	Thermal conductivity	[W/mK]
x,y	:	Spatial coordinates	[m]
q	:	Heat flux in/out	[W/m^2]
N	:	Element interpolation polynom	

The interpolation polynom **N** is defined as follows:

$$N_1 = \frac{1}{4}(1+\xi)(1+\eta)$$

$$N_2 = \frac{1}{4}(1-\xi)(1+\eta)$$

$$N_3 = \frac{1}{4}(1-\xi)(1-\eta)$$

$$N_4 = \frac{1}{4}(1+\xi)(1-\eta)$$

which gives a linear variation over the element.
Temperature variation over the element thickness is disregarded.
ξ and η are the **non-dimensional** element coordinates ranging from -1 to 1.
The temperature field within the element is defined as follows:

$$T(\xi,\eta) = \mathbf{N}\,\mathbf{T}$$

where **T** is the nodal temperature vector.

The element coordinates within the element are defined similarly:

$$x(\xi,\eta) = \mathbf{N}\,\mathbf{x}$$
$$y(\xi,\eta) = \mathbf{N}\,\mathbf{y}$$

In connection with the integration over the element volume, it is of interest to express the element matrices and vectors as a function of the non-dimensional variables ξ and η only.

The derivation: $\frac{\partial \boldsymbol{N}}{\partial x}$, $\frac{\partial \boldsymbol{N}}{\partial y}$ are used in the expression for the conductivity matrix.

This derivation with respect on x and y should be transferred to derivation with respect on ξ and η:

Partial derivation presented on matrix form:

$$\begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} = \boldsymbol{J} \cdot \boldsymbol{\Delta}_{x,y}$$

where **J** is the **Jacobian matrix.**

The inverse relationship is then obtained as follows:

$$\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} = \frac{1}{\det \boldsymbol{J}} \begin{bmatrix} \frac{\partial y}{\partial \eta} & -\frac{\partial y}{\partial \xi} \\ -\frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \xi} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} = \boldsymbol{J}^{-1} \cdot \boldsymbol{\Delta}_{\xi,\eta}$$

The derivation with respect on **x** may then be expressed as follows:

$$\boldsymbol{N}_{,x} = \frac{1}{\det \boldsymbol{J}} \left(\frac{\partial y}{\partial \eta} \boldsymbol{N}_{,\xi} - \frac{\partial y}{\partial \xi} \boldsymbol{N}_{,\eta} \right)$$

Similar expressions are obtained when derivation with respect on **y**.

The volume integral over the element expressed in ξ and η :

$$\int_V dV = \int_{-1}^{1} \int_{-1}^{1} h(\xi,\eta)\, dA = \int_{-1}^{1} \int_{-1}^{1} h(\xi,\eta) \det \boldsymbol{J}\, d\xi\, d\eta$$

where h(ξ,η) = **N h** is the element thickness which may vary over the element and **h** is the element thickness at the corners.

The element matrices are integrated numerically using **Gaussian integration**.

3.3.2 Energy exchange

The netto element heat flux, q, has contribution from several different terms:

Elements with 1 outside and 1 inside, (hollow profiles):

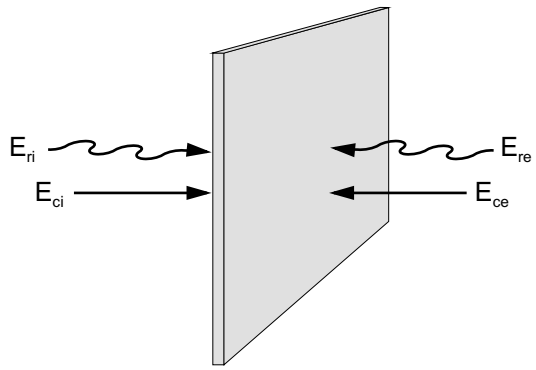


Figure 3.3.2.1 Energy exchange calculation terms at non-insulated surface, inside + outside

For non-insulated elements the following terms contribute to the resultant element heat flux:

$$q = E_{re} + E_{ce} + E_{ri} + E_{ci}$$

where:

E_{re} : Netto surface radiant energy input at the **exterior** side of the element
E_{ri} : Netto surface radiant energy input at the **interior** side of the element
E_{ce} : Netto surface convection energy input at the **exterior** side of the element
E_{ci} : Netto surface convection energy input at the **interior** side of the element

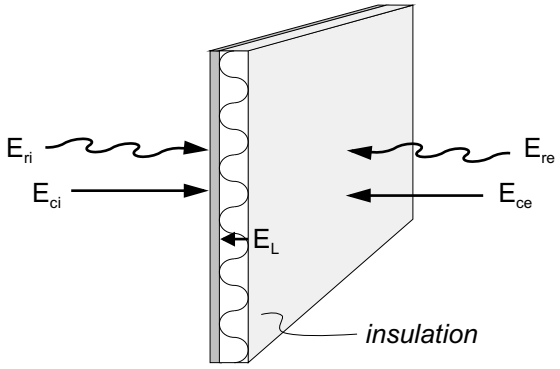


Figure 3.3.2.2 Energy exchange calculation terms at insulated surface, inside + outside

For insulated elements the following terms contribute to the resultant element heat flux:

$$q = E_l + E_{ri} + E_{ci}$$

where:

E_l : Need energy 'leakage' through the insulation

Elements with 2 outsides, (open profiles):

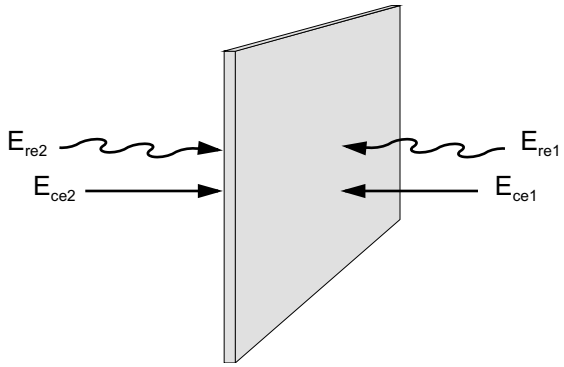


Figure 3.3.2.3 Energy exchange calculation terms at non-insulated surface, 2 outsides

For non-insulated elements the following terms are contributing to the resultant element heat flux:

$$\boldsymbol{q} = \boldsymbol{E_{re1}} + \boldsymbol{E_{ce1}} + \boldsymbol{E_{re2}} + \boldsymbol{E_{ce2}}$$

where:

E_{re1} : Need surface radiant energy input at the **exterior** side 1 of the element
E_{re2} : Need surface radiant energy input at the **exterior** side 2 of the element
E_{ce1} : Need surface convection energy input at the **exterior** side 1 of the element
E_{ce2} : Need surface convection energy input at the **exterior** side 2 of the element

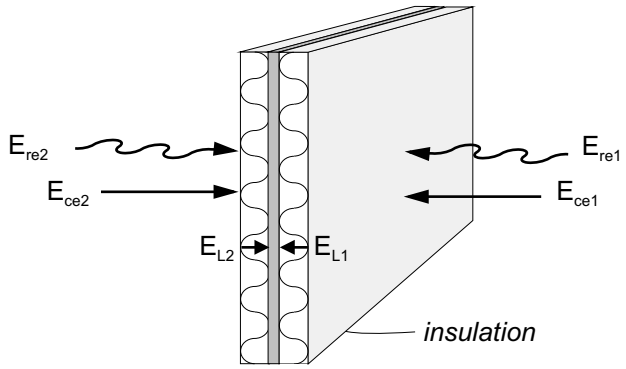


Figure 3.3.2.4 Energy exchange calculation terms at insulated surface, 2 outsides

For insulated elements the following terms are contributing to the resultant element heat flux:

$$\boldsymbol{q} = \boldsymbol{E_{l1}} + \boldsymbol{E_{l2}}$$

where:

E_{l1} : Need energy 'leakage' through the insulation at side 1 of the element
E_{l2} : Need energy 'leakage' through the insulation at side 2 of the element

3.3.3 Insulation

The quadrilateral heat transfer element may be 'protected' by thermal insulation on one or both surfaces. In connection with the heat transfer calculations, **the resultant energy 'leakage'** through the insulation is of interest.

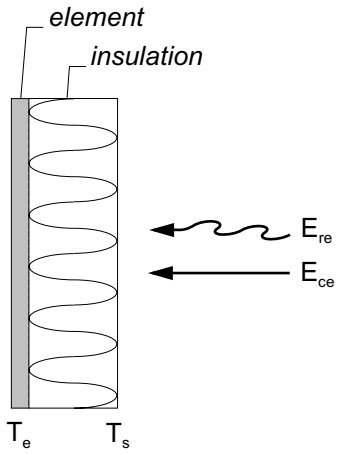


Figure 3.3.3.1 Insulation Boundary Conditions

Temperatures within the insulation are not present in the system equations, each insulation element has its own separated equation system which may not be a linear system.

It is assumed that the heat flow through the insulation is 1 dimensional because the thickness of the insulation is of an order smaller than the outstretch in the surface directions.

Further, it is assumed that the temperatures in the main structure increase slower than the exposed side of the insulation, (that is why the insulation is present), The main structure temperatures from the previous incremental solution represents then a fixed boundary condition at the interior side of the insulation.

The resultant energy 'leakage' trough the insulation is expressed as follows:

$$E_l = f(T_e, T_s, E_{re}, E_{ce})$$

where:

T_e : Basic element temperature
T_s : Insulation surface temperature

The need surface energy input is dependent on T_s, and it is then necessary to solve a fourth order equation which includes radiation energy balance, convection energy balance and heat transfer through the insulation itself, (conduction, accumulation).

MASSLESS, TEMPERATURE DEPENDENT INSULATION, (type 1):

The heat transfer through the insulation is defined as follows:

$$E_l = K(T)(T_e - T_s)$$

where:

K(T) : Resultant thermal conduction

The characteristic temperature is : $T = \frac{1}{2}(T_e + T_s)$ and this temperature is used to find the current resultant thermal conduction.

If the insulation is exposed to f.inst. a standard fire, (gas with a prescribed temperature/time development), the heat transfer through the insulation is calculated as follows:

$$E_l = E_s + E_c$$

inserted for expressions for the heat transfer through the insulation, surface radiant energy balance and surface convection energy balance:

$$K(T_e - T_s) = \sigma\epsilon_s(T_s^4 - \epsilon_g T_g^4) + C(T_s - T_g)$$

which gives the following 4'th degree equation to find the insulation surface temperature T_s :

$$\sigma\epsilon_s T_s^4 + (K+C)T_s - (KT_e + \sigma\epsilon_s\epsilon_g T_g^4 + CT_g) = 0$$

When the surface temperature is found the heat transfer through the insulation is known.

3.3.4 View Factors

The view factor, $\mathbf{F_{ij}}$, is defined at the fraction of total radiant energy that leaves surface **'i'** which arrives directly on surface **'j'**, see figure 3.3.4.1.

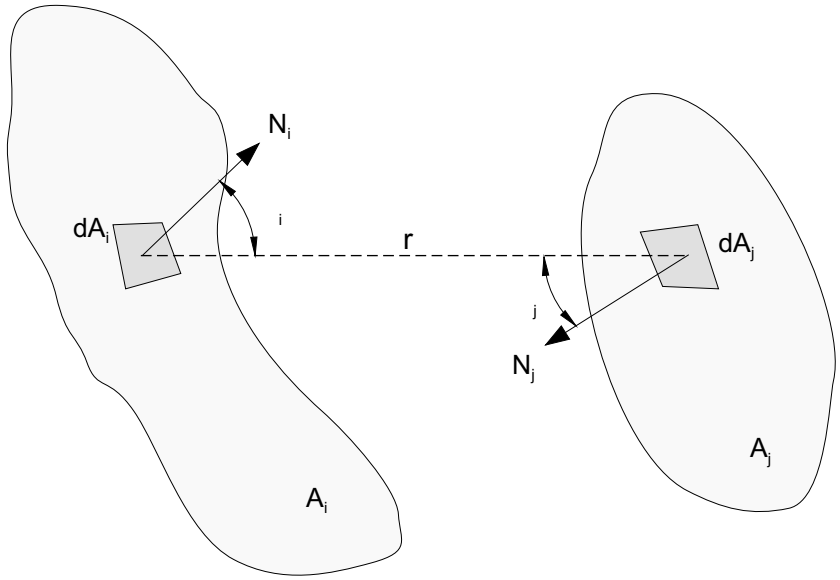


Figure 3.3.4.1 View Factor Calculation Terms

The view factor is expressed in the following equation:

$$\boldsymbol{F_{ij}} = \frac{\mathbf{1}}{\boldsymbol{A_i}} \iint_{A_i A_j} \frac{\mathbf{\cos\theta_i \cos\theta_j}}{\boldsymbol{\pi r^2}} \, \boldsymbol{dA_j dA_i}$$

Where

A_i ,A_j : Area of surface i and j
r : Distance between differential surfaces i and j
θ_i : Angle between surface normal, $\mathbf{N_i}$, and the radius line to surface dA_j
θ_j : Angle between surface normal, $\mathbf{N_j}$, and the radius line to surface dA_i
$\mathbf{N_i}$,$\mathbf{N_j}$: Surface Normal of differential surfaces dA_i and dA_j

If the areas A_i and A_j are small compared with the distance r between them, the following simplified equation may be used:

$$\boldsymbol{F_{ij}} = \frac{\mathbf{A_j \cos\theta_i \cos\theta_j}}{\boldsymbol{\pi r^2}}$$

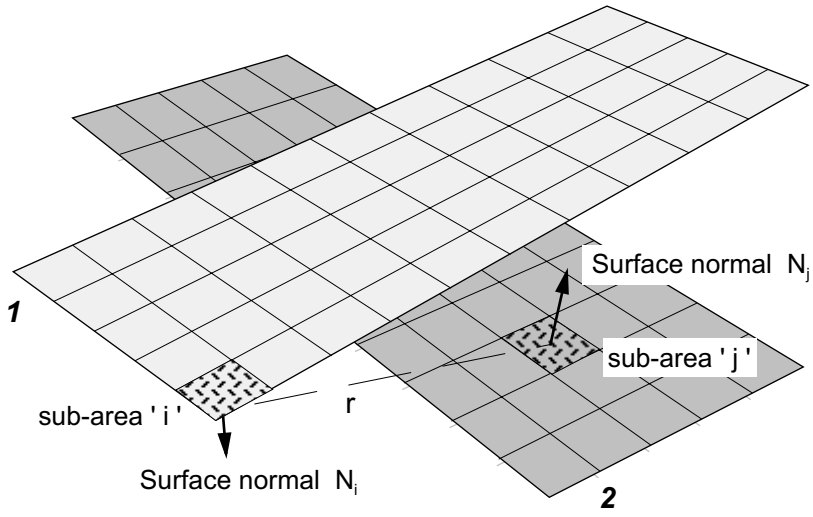


Figure 3.3.4.2 Numerical Integration approach for View factor calculations

Figure 3.3.4.2 describes two arbitrarily shaped quadrilaterals with area A_1 and A_2.
Area 1 is sub-divided into **'n'** small enough areas for which the **simplified view factor calculation are valid**. Similarly, area 2 is sub-divided into 'm' small areas.

The view factor $\mathbf{F_{12}}$ is calculated as follows:

$$F_{12} = \frac{1}{A_1} \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} A_i A_j$$

Where

A_1 : Area of quadrilateral 1
A_i ,A_j : Area of sub-areas i and j
r : Distance between sub area surfaces i and j
θ_i : Angle between surface normal, $\mathbf{N_i}$, and the radius line to surface dA_j
θ_j : Angle between surface normal, $\mathbf{N_j}$, and the radius line to surface dA_i
$\mathbf{N_i}$,$\mathbf{N_j}$: Surface Normal of sub area surfaces A_i and A_j

3.3.5 Heat Accumulation Element

To account for thermal energy accumulation or the 'inertia' effect due to the presence of a gas or a fluid inside hollow profiles, a simple 'Heat Accumulation Element' is introduced.

The element has no conduction terms, only a lumped mass matrix, (diagonal matrix), is established for this element.

The heat capacity is calculated as follows:

$$m_i = \frac{A_i l \rho_i}{n}$$

where

m_i : Lumped mass at node i
A_i : Internal cross section area
l : Length of the member
ρ_i : Mass density of the interior fluid
n : Number of element nodes

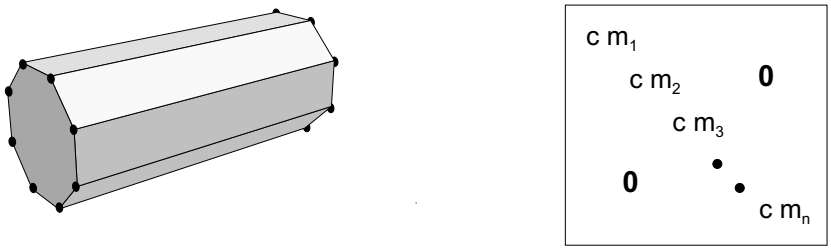


Figure 3.3.5.1 Heat Accumulation Element Mass Matrix

3.4 TEMPERATURE ANALYSIS MODEL FOR STRUCTURAL MEMBERS

3.4.1 Profile Types

TUBULAR MEMBERS

Tubular members are meshed as shown in figure 3.4.1.1.
The mesh refinement in the length- and circumferential directions are controlled by the user.

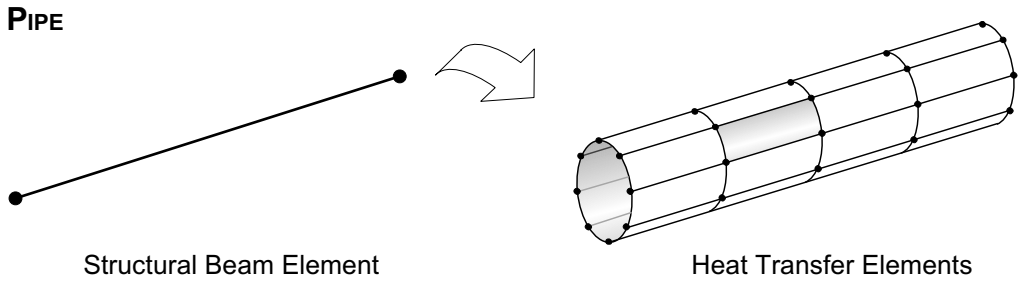


Figure 3.4.1.1 Heat transfer element mesh for tubular members

The heat transfer elements belonging to tubular members have 1 inside and 1 outside.

Only the outside is exposed to the actual fire scenario.

If insulation is prescribed for tubular members, only the outside surface of the heat transfer elements are 'covered' with the actual thermal protection.

'Heat accumulation element' is generated since the profile is hollow.

RECTANGULAR HOLLOW SECTIONS (RHS/BOX)

RHS members are meshed as shown in figure 3.4.1.2.
The mesh refinement in the upper flange, 'webs' and the lower flange are controlled by the user.

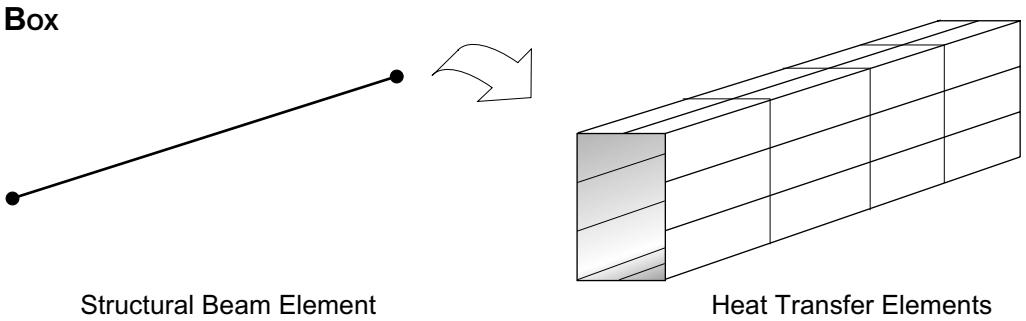


Figure 3.4.1.2 Heat transfer element mesh for Rectangular Hollow Sections

The heat transfer elements belonging to box profiles have 1 inside and 1 outside.

Only the outside is exposed to the actual fire scenario.

If insulation is prescribed for box members, only the outside surface of the heat transfer elements are 'covered' with the actual thermal protection.

'Heat accumulation element' is generated since the profile is hollow.

I/H PROFILES

I/H profiles are meshed as shown in figure 3.4.1.3.
The mesh refinement in the upper flange, web and the lower flange are controlled by the user.

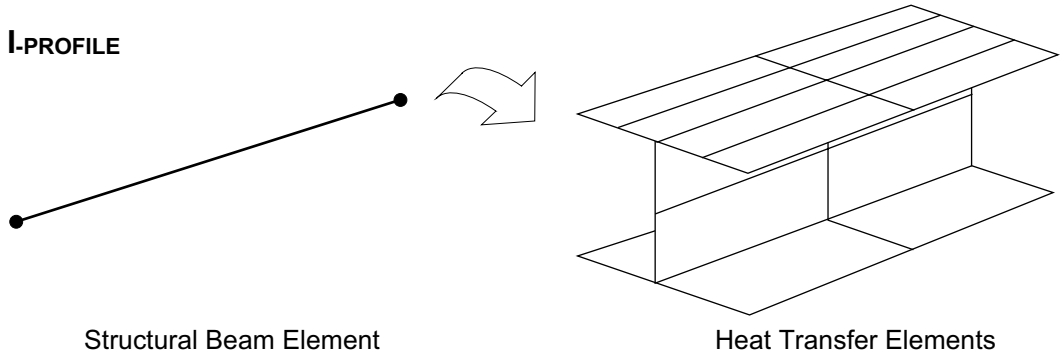


Figure 3.4.1.3 Heat transfer element mesh for I/H - Sections

The heat transfer elements belonging to I/H profiles have 2 outsides.

Both outsides are exposed to the actual fire scenario.

If insulation is prescribed for I/H members, both surfaces of the heat transfer elements are 'covered' with the actual thermal protection.

'Heat accumulation element' is **not** generated since the profile is open.

PLATE / SHELL

4 node plate- and shell elements are meshed as shown in figure 3.4.1.4.
The mesh refinement in the two directions are controlled by the user.

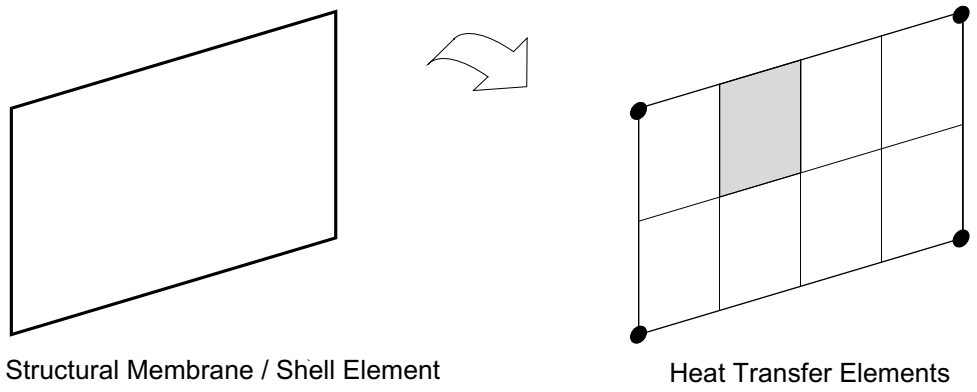


Figure 3.4.1.4 Heat transfer element mesh for 4 node plate and shell elements

The heat transfer elements belonging to plate/shell elements have 2 outsides.

Both outsides are exposed to the actual fire scenario.

If insulation is prescribed for plate/shell elements, both surfaces of the heat transfer elements are 'covered' with the actual thermal protection.

'Heat accumulation element' is **not** generated since the element is open.

3.4.2 Linearization of temperature field

In the integrated system for fire analysis the output from the temperature analysis module will represent a general non-uniform temperature distribution in the structure considered. Depending on the heating conditions, structural geometry and insulation, complex temperature configurations may occur.

As the response calculations in USFOS is based on a simplified linear temperature distribution, a transformation of the "true" temperature state is required prior to the structural response calculation, in order to represent the thermal expansion forces in a realistic manner.

This section discusses a simple way to process the results from a detailed temperature analysis, and produce a temperature load file which is input to USFOS for the collapse analysis.

The cross section temperature distribution may be expressed as:

$$t(x,y,z) = t_0 + \bar{t}(x,y,z) = t_0 + \beta_y \cdot z + \beta_z \cdot y$$

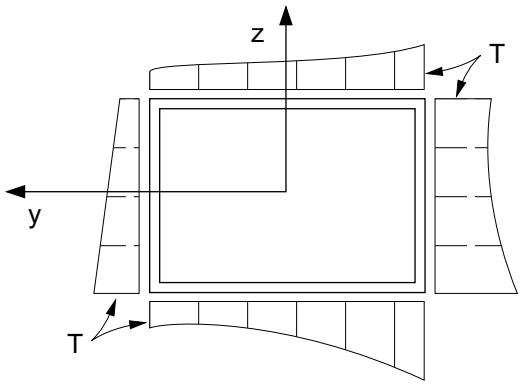


Figure 3.4.2.1 Cross section temperature distribution

The proposed approach is to find an equivalent temperature state, characterized by the parameters ΔT_0, $\Delta\beta_y$, $\Delta\beta_y$, which produces the same increment in thermal expansion forces as the true temperature distribution calculated by FAHTS.

The equivalent nodal expansion forces for the beam element due to the incremental temperature change is by use of the potential energy approach found as /USFOS Theory/:

$$\int_0^L \int_A E((-\alpha\Delta t)(\delta u_{,x} + v_{,x}\delta v_{,x} + w_{,x}\delta w_{,x})$$
$$+ \int_0^L \int_A E\alpha\Delta t_y dA\,\delta v_{,xx} dx$$
$$+ \int_0^L \int_A E\alpha\Delta t_z dA\,\delta w_{,xx} dx$$

The calculation of the equivalent temperature change is based on the following equations:

$$E_s I_z \Delta\beta \int_0^L \mathbf{N}^{\theta}{}_{,xx}\, dx = \int_0^L \int_A E(x,y,z)\,\Delta t(x,y,z)\cdot y \cdot \mathbf{N}^{\theta}{}_{,xx}\, dA\, dx$$

$$E_s I_y \Delta\beta \int_0^L \mathbf{N}^{\theta}{}_{,xx}\, dx = \int_0^L \int_A E(x,y,z)\,\Delta t(x,y,z)\cdot z \cdot \mathbf{N}^{\theta}{}_{,xx}\, dA\, dx$$

$$E_s A \Delta\beta \int_0^L \mathbf{N}^{u}{}_{,x}\, dx = \int_0^L \int_A E(x,y,z)\,\Delta t(x,y,z)\cdot \mathbf{N}^{u}{}_{,x}\, dA\, dx$$

From these equations the equivalent $\boldsymbol{\Delta\beta_{y\Delta}}$, $\boldsymbol{\Delta\beta_{z\Delta}}$ **and** $\boldsymbol{\Delta T_0}$ may be calculated.

N denotes the beam interpolation function.

3.5 HEAT SOURCES

3.5.1 Element Flux

Prescribed element flux may be defined by specifying a heat flux to be applied to all heat transfer element belonging to the specified structural element(s).

The heat flux is specified in the actual units.

3.5.2 Nodal Boundary Temperature

Prescribed (forced) node temperatures may be defined by specifying the actual temperature to be applied to the **structural node(s)**. All nodes in the heat transfer model belonging to elements connected to the defined node(s) are given the prescribed temperature which is kept constant through the simulation.
In figure 3.5.2.1 the marked nodes in the heat transfer model will be assigned the prescribed temperature of the structural node no. 1.

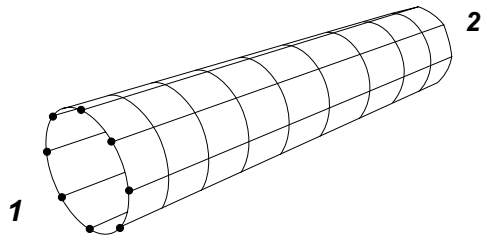


Figure 3.5.2.1
Prescribed node temperatures

3.5.3 Time dependent Environmental temperature

By specifying a time dependent environmental gas temperature, the user may define a specific fire scenario.
All surfaces, (outsides) of the heat transfer elements will be exposed to this heat 'loading'.
The heat fluxes are calculated according to Sect. 3.2.4.
The emissivity coefficient of the gas is set equal to 1.

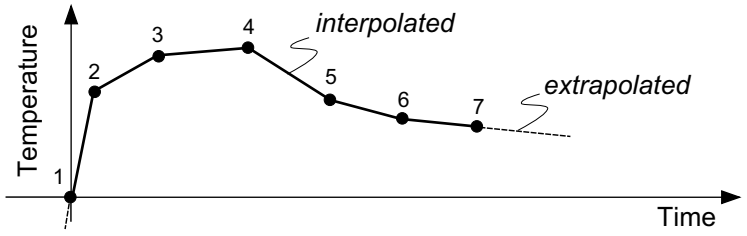


Figure 3.5.3.1 Time dependent environmental temperature

3.5.4 Time dependent Concentrated Source

The user may apply a prescribed concentrated source defined by its location and energy emittance.
The heat rays are emitted in all directions, and the heat flux received by the individual heat transfer elements is calculated on basis on:

$$q_i = \frac{E}{4\pi r_i^2}\cos(\theta_i)$$

where

E : Total energy emittance from the source
ri : Distance between source and element midpoint
θ_i : Angle between the ray and the element surface normal

The source may be defined **time dependent**.

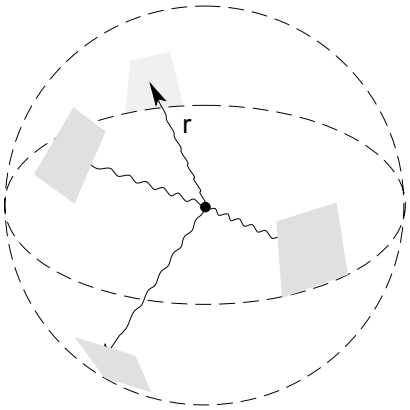


Figure 3.5.4.1 Time Dependent Concentrated Source

3.5.5 Time dependent Line Source

The user may specify a line-source defined by its location, shape and energy emittance.

The global coordinates of 'end' 1 and 'end' 2 are given together with the energy emittance (J/m) at the two ends which may be different. The shape is defined by the diameter of the 'cylinder/cone' at end 1 and end 2.

The line source is simulated by **n** discrete sources each with energy emittance:

ΔE = ΔL * e

where:

ΔL : Distance between discrete sources = L/(n-1)
e : Specified energy emittance per length unit

The end sources are emitting 50% less energy than the rest.

Contribution from all **n** sources are calculated similar to the **Concentrated Source**, (see sect 3.5.4) and accumulated as follows:

$$Q = \sum_{i=1}^{n} q_i(E_i, r_i, \theta_i)$$

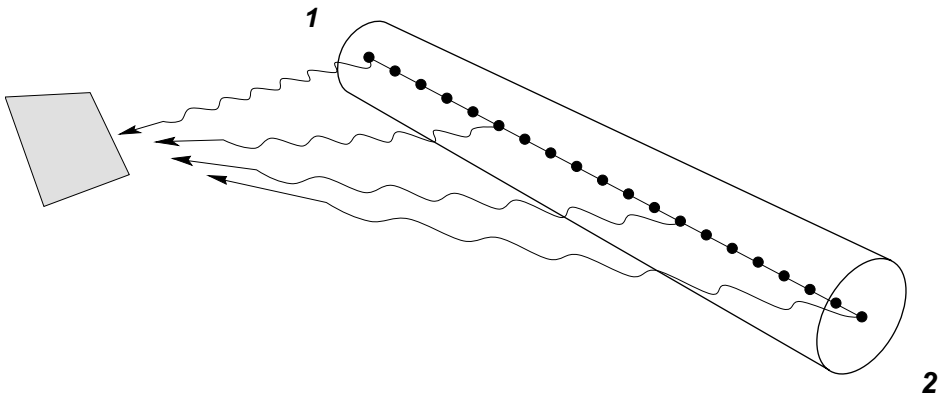


Figure 3.5.5.1 Line source simulated by discrete sources

If a heat transfer elements is located **inside** the cylinder-surface, the element is assigned a flux with same intensity a the flux at the cylinder-surface. No reduction of the flux due to surface orientation vs. location of the source is accounted for, (cos(θ) is set equal to 1.0).

The line-source may be defined **time dependent**.

3.5.6 KAMELEON / FIREINT interface

Each finite element in the heat transfer model are treated individually by the KAMELEON / FIREINT interface.
Based on the element mid coordinates, the surface normal, the current surface temperature and the thermal properties of the surface, the interface routine calculates the resulting heat flux.

The finite element has to be located inside the calculation domain of the fire simulation. The computational domain grid, gas velocities, temperatures and the gas absorption coefficients are read from files written by the fire simulation program.

The gas absorption coefficients are computed by the fire simulation code based on the concentrations of CO_2, H_2O and soot and by the current gas temperature.

These files form a transient development of the fire, and the file which is nearest in time is chosen. This means that the last file is chosen for simulation time exceeding the actual fire simulation.

Radiation:

The incoming radiation is based on "**The Discrete Transfer Method for Radiation" by Shah and Lockwood**. A number of rays are sent from the surface of the element and are followed until they hit the opposite boundary of the calculation domain. It is given the radiation intensity defined by the temperature and the absorption coefficient of the wall point and followed back to the element surface.
The radiation intensity is increased or decreased based on the conditions in the control volumes the ray passes through on its way back to the element.

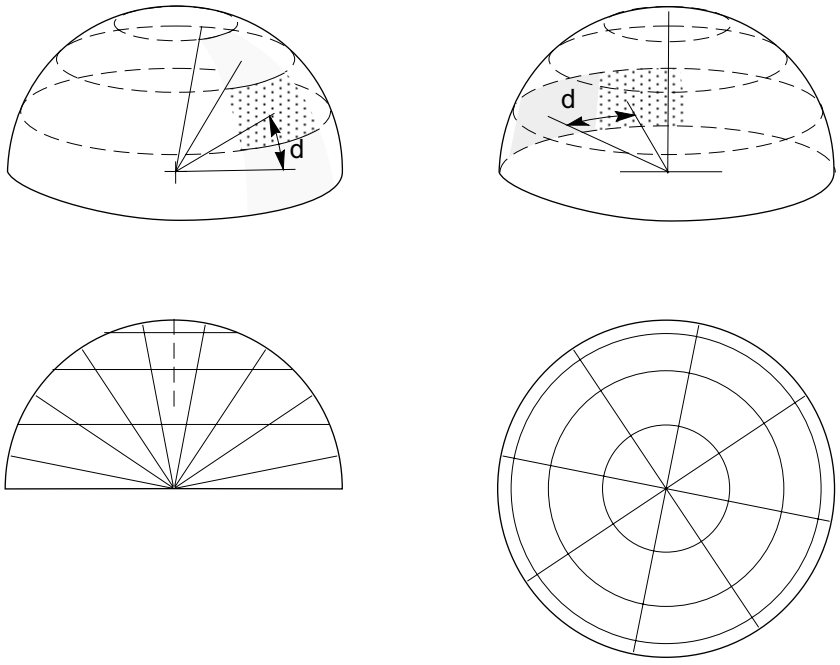


Figure 3.5.6.1 Discrete rays, definition of nθ and nΦ

The rays are distributed through a semi-sphere, where the midpoint of the element is located in the center. The semi sphere is subdivided by dividing the angle between the element and its normal vector. This forms a number of circular lines on the semi-sphere parallel to the element surface. Each of these lines are split into equal parts. Each ray passes through one of the points defined by this method.
Each ray represent a discrete angle. This means that a point far away from the element which is hit represents a larger area than a closer point. Because of this, the number of rays should be increased if the flame is located far away from the element. If the number of rays is small, there is a large risk that the flame is not hit - this means that the element will not be influenced by the flam at all - or that a small, very hot part of the flame is hit, over-estimating the influence of the heat radiation.

The number of rays is governed by a parameter "**nAccur**". This is split into the numbers **nθ**, which is the number of subdivisions of the angle between the element surface and its normal, and **nΦ**, which is the number of subdivisions of the circles parallel to the element surface, see figure 3.5.6.1. The formulas: **nθ = 3 * nAccur ,and nΦ = 4 * nAccur** are used, (nAccur is input from the user).

This means that the number of rays varies from 12, (nAccur = 1, minimum value),
to 4800, (nAccur = 20, maximum value).

The subdivisions, nθ and nΦ are defined in figure 3.5.6.1. From the figure it is seen that the accuracy decreases with increasing distance between the element and the radiating source.

The outgoing radiation is computed based on the element's current temperature and emissivity coefficient.

Convection

The convection is based on the temperature and the velocity of the gas outside the element, and on the temperature of the element's surface. The velocity is found as the vector sum of the velocity components in the three directions. The temperature and velocity are taken from the control volume in which the mid-point of the element is located, i.e. no interpolation takes place.

The convective heat transfer coefficient, H_c is computed according to the formula:

$$\mathbf{H_c = N_u K_l / L}$$

where

N_u : Nusselt number
K_l : Thermal conductivity of air
L : Characteristic length scale (set to 0.2)

The Nusselt number is base on a formula involving local Reynolds number and the Prandtl number (set to 0.707).

3.5.7 ISO - fire

By specifying a ISO- fire a prescribed temperature development of the surrounding gas is applied according to ISO-834.
All surfaces, (outsides) of the heat transfer elements will be exposed to this heat 'loading'.
The heat fluxes are calculated according to Sect. 3.2.4.
The emissivity coefficient of the gas is set equal to 1.

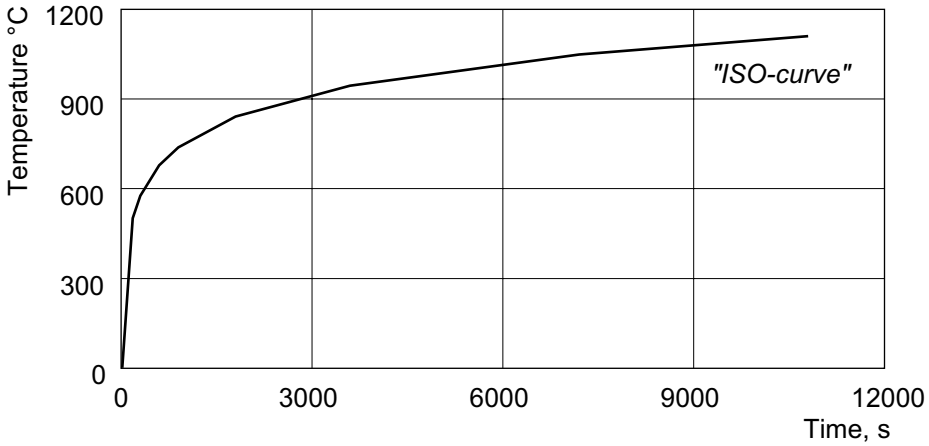


Figure 3.5.7.1 Gas temperature development according to ISO-834

3.5.8 HC - fire

By specifying a HC- fire a prescribed temperature development of the surrounding gas is applied according to a **standard Hydro Carbon fire defined by NPD**.
All surfaces, (outsides) of the heat transfer elements will be exposed to this heat 'loading'.
The heat fluxes are calculated according to Sect. 3.2.4.
The emissivity coefficient of the gas is set equal to 1.

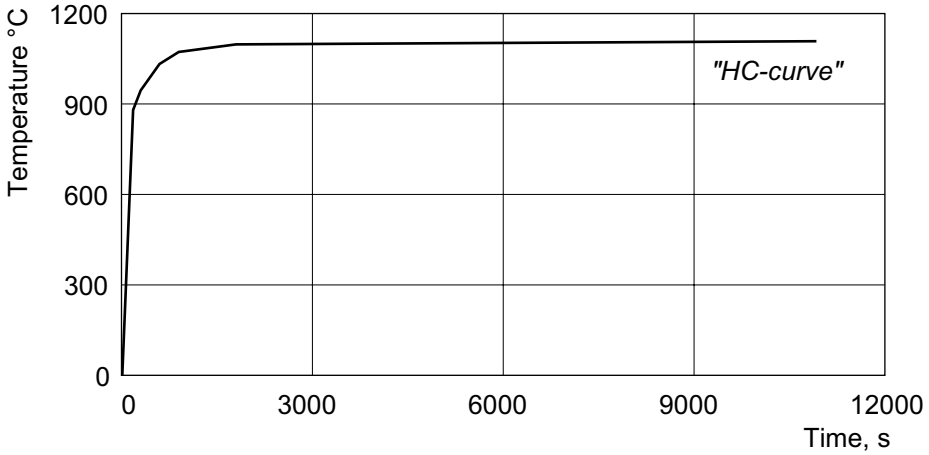


Figure 3.5.8.1 Standard Hydro Carbon Fire temperature development